Pie Mathematics Association

$e^{i\pi}+1=0$

We Serve Who Deserve

Pie Mathematics Association

(Registered Trust)

Mathematical Pixels

(e-Magazine for Enrichment of Mathematics)

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Editor

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Editor's Foreword

In view of propagating and popularizing mathematics, Pie Mathematics Association (PMA) has been involved in various activities and outreach programmes. The association has been publishing one book per year since its inception in view of reaching several mathematics enthusiasts and professionals. In the year 2020, when the whole world have to rely on online stream due to pandemic, PMA decided to release e-magazine for meeting our objective of spreading mathematics among everyone. This has resulted in publishing this first issue of e – magazine titled "Mathematical Pixels". I thank Sri. S. Manooj Kumar for suggesting wonderful title to this magazine. My sincere thanks to the trustee Dr. P.N. Vijayakumar for helping us in making this e-magazine.

The purpose of publishing this e – magazine is to encourage and motivate young students and interested teachers who need to know mathematics beyond their usual curriculum. I hope that this magazine will satisfy this need. We also encourage everyone interested in mathematics to send their articles for future publication in the mail id piemathematicians@yahoo.com Depending on the quality and innovativeness, the editorial team may decide to publish. In this first issue, we have published six expository articles. On behalf of PMA, I thank everyone who have been responsible for publishing this issue. PMA welcome constructive suggestions for improvement of the magazine.

We are pleased to inform that the first issue of the magazine was released online by the distinguished mathematician Professor M. Ram Murty on 19th December, 2020 during the occasion of centenary remembrance year and 133rd Birthday celebration of Srinivasa Ramanujan.

Dr. R. Sivaraman

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Quadratic Sequence and its Applications

Prof. J. Suganthi

Introduction

Sequence and Series of real numbers are one of the important concepts in mathematics. A quadratic sequence is a sequence of numbers in which the second differences between consecutive terms of the given sequence is always constant. That constant is called a common second difference. The purpose of this article is to understand the properties and applications of Quadratic sequences.

Keywords: First difference, Sequence, Common second difference, General Term.

1. Definition

Let $a_1, a_2, a_3, \ldots$ be given sequence of real numbers. The differences between consecutive terms namely

$$a_2 - a_1, a_3 - a_2, a_4 - a_3, a_5 - a_4, \ldots$$

are called first differences. If we assume $b_k = a_{k+1} - a_k$ then the sequence $b_1, b_2, b_3, \ldots$ is called sequence of first differences of $a_1, a_2, a_3, \ldots$

We now calculate the difference between consecutive terms of the sequence of first differences namely

$$b_2 - b_1, b_3 - b_2, b_4 - b_3, b_5 - b_4, \ldots$$

If these difference values happens to be constant, that is, if $b_{k+1} - b_k = d$ where d is a constant, then the given sequence $a_1, a_2, a_3, \ldots$ is called a quadratic sequence.

2. Illustration

Consider an example of a sequence 1, 2, 4, 7, 11, . . .

We claim that 1, 2, 4, 7, 11,... is a quadratic sequence because

If we take the first difference between consecutive terms then

$$a_2 - a_1 = 2 - 1 = 1$$
$$a_3 - a_2 = 4 - 2 = 2$$
$$a_4 - a_3 = 7 - 4 = 3$$
$$a_5 - a_4 = 11 - 7 = 4$$
....

The sequence of first differences is thus 1,2,3,4,...
We now try to calculate the second difference, which is simply obtained by taking the difference between the consecutive terms of 1,2,3,4,... we get

$$b_2 - b_1 = 2 - 1 = 1$$
$$b_3 - b_2 = 3 - 2 = 1$$
$$b_4 - b_3 = 4 - 3 = 1$$
$$b_5 - b_4 = 5 - 4 = 1$$
....

We notice that the second differences are all equal to 1. Thus by definition, the sequence 1, 2, 4, 7, 11, . . . is a quadratic sequence.

3. General Case

If the given sequence is quadratic, its *n*th term should be of the form

$$T_n = An^2 + Bn + C \quad (1)$$

Using (1), we now construct the following tabular values

n	1	2	3	4	5
Terms	A+B+C	4A+2B+C	9A+3B+C	16A+4B+C	25A+5B+C
1st difference		3A+B	5A+B	7A+B	9A+B
2nd difference			2A	2A	2A

In this case, the second difference values are always 2A. This fact can be used to determine A, B, C, the constants making up the quadratic sequence.

4. Determining nth – term of a Quadratic sequence

Let the *n*th term for a quadratic sequence be given by equation $T_n = An^2 + Bn + C$ where A, B, C are some constants to be determined.

$$T_1 = A(1)^2 + B(1) + C = A + B + C$$
$$T_2 = A(2)^2 + B(2) + C = 4A + 2B + C$$
$$T_3 = A(3)^2 + B(3) + C = 9A + 4B + C$$

Let $d = T_2 - T_1$ Then $d = (4A + 2B + C) - (A + B + C) = 3A + B.$

$$B = d - 3A \quad ....(1)$$

Let D be the second common difference of the given quadratic sequence.

The common second difference is obtained from $D = (T_3 - T_2) - (T_2 - T_1)$

$$D = (5A+2B) - (3A+B) = 2A$$

From (1) $$A = \frac{D}{2}, \quad B = d - \frac{3D}{2} \quad ....(2)$$

$$C = T_1 - (A + B) = T_1 - \left(\frac{D}{2} + \left(d - \frac{3D}{2}\right)\right)$$

$$C = T_1 + D - d \quad ....(3)$$

Finally, the general equation for the *n*th term of a quadratic sequence from (2) and (3) is given by

$$T_n = \left(\frac{D}{2}\right)n^2 + \left(d - \frac{3D}{2}\right)n + (T_1 - d + D) \quad ....(4)$$

As an example, for the quadratic sequence in our illustration given by 1, 2, 4, 7, 11, . . .

we have $d = 2-1 = 1$, D = second difference = 1.

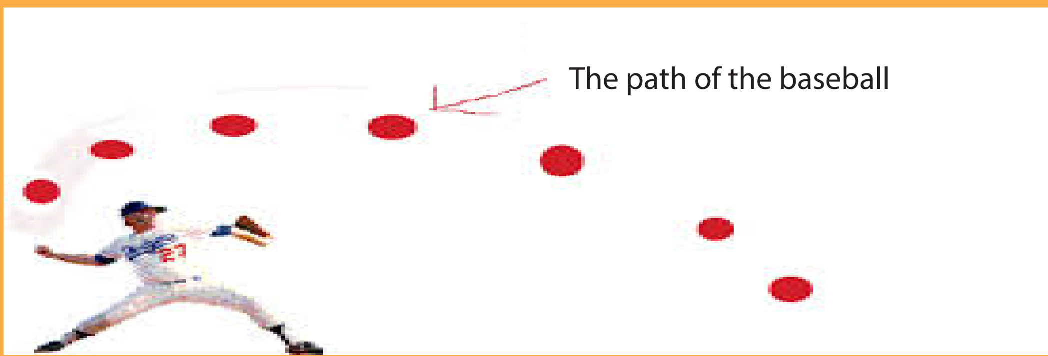
Thus from equation (4) we have

$$T_n = \left(\frac{1}{2}\right)n^2 + \left(-\frac{1}{2}\right)n + (1-1+1) = \frac{n^2 - n + 2}{2}$$

5. Applications

Here are couple of practical applications of quadratic sequence.

5.1 The distance a baseball travels depends on the angle at which it is hit and its speed. The table shows the distances a baseball that is hit at an angle of 40^0 travels at various speeds at different distances that it travels. Describe the pattern of the distances traveled by the baseball.



Speed(mph)	80	85	90	95	100	105	110	115
Distance(ft)	194	220	247	275	304	334	365	397

Solution: We will find the successive differences of the entries in the given tabular column.

194 220 247 275 304 334 365 397

26 27 28 29 30 31 32

1 1 1 1 1 1

Because the second differences are constant, the pattern of the distances traveled by baseball follows a quadratic sequence.

5.2 Let us consider an experiment to determine the motion of a free falling object. You drop a shot-put ball from a height of 256 feet and measure the distance it has fallen at various times.

Time(sec)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Distance(ft)	0	1.75	5	9.75	16	23.75	33	43.75	56

Is the pattern of the distances quadratic?

Solution Let us consider the difference table of distances that the shot-put ball has traveled.

0	1.75	5	9.75	16	23.75	33	43.75	56
	1.75	3.25	4.75	6.25	7.75	9.25	10.75	12.25
		1.5	1.5	1.5	1.5	1.5	1.5	1.5

We notice that the second difference values are all 1.5 which is constant. Hence the distance pattern follows a quadratic sequence.

Conclusion

It is incredible to note how one sequence of numbers can be applied to various real-life situations proving the utility of mathematics. I personally felt excited to write this article and thank Pie Mathematics Association for providing this wonderful opportunity.

Prof. J. Suganthi was Head of Department of Mathematics at S.S.K.V. College of Arts & Science for women, Kanchipuram,Tamil Nadu. She has more than 15 years of experience of teaching at school and college level. Learning deeply and teaching interestingly was her passion. She has written some articles for magazines and had been involved in the activities of propagating mathematics.

Do We Sleep Mathematically?

Dr. P. N. Vijayakumar

Introduction

By what time we feel brisk after we wake up in the morning? Mathematics plays a vital role in answering this question. This article is focussed about explaining the connection of sleeping hours of humans with mathematics. Do we find any pattern behind this? Let's proceed further to know.

Keywords: **Sleep Cycle, REM and Non – REM Sleeps, Periodic Function, Sleep Cycle Length.**

1. Modeling Sleep Cycle

Usually our body repair themselves while sleeping. So, sleeping is very essential in our day to day life. Does mathematics have anything to do with sleep cycle? Generally, if we sleep eight hours per day then it will be good for our health. We will understand the mechanism of sleep cycle by describing it through a mathematical function.

2. Describing Sleep hours mathematically

Upon analyzing the sleep mechanism of humans, we find that sleep occurs in different stages. Our typical sleep cycle begins with what is known as REM sleep. (REM refers to Rapid Eye Movement). The two main categories of sleep are REM sleep and Non – REM sleep. Typically REM sleep category doesn't produce deep sleep process. Our body will be at rest, but our mind would be active to certain extent. Dreams usually occur in REM sleep category. In Non – REM sleep category, there are four stages. Our body repairs itself only in this category.

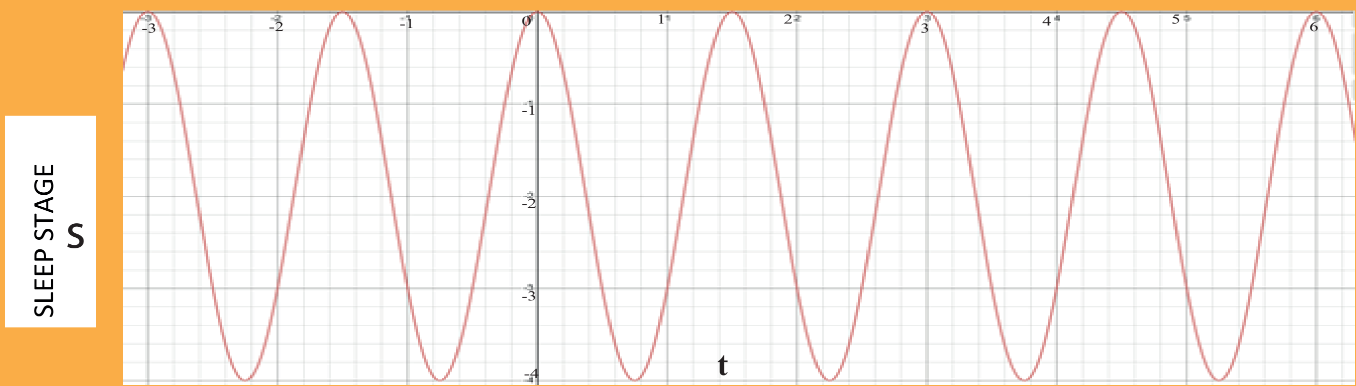


Figure 1

In Figure 1, we denote stages of sleep as 'S' along Y – axis and time duration of sleep in hours as 't' along X – axis. Since sleep can be categorized depending upon the number of hours slept, S depends on t. Thus we consider S as a function of t, where t is the number of hours slept. That is, S = f (t).

From Figure 1, we observe that the function representing sleep pattern repeats for every 1.5 hours. Such a function is called as periodic function. In General, periodic functions are those whose values repeat after an fixed interval of time T. The number T is called its period. We can notice that the graph in Figure 1, behave same way in the intervals [–3, –1.5], [–1.5,0], [0,1.5], [1.5,3], [3,4.5]. So the function representing stages of sleep is a periodic function with period 1.5 hours. Thus, Figure 1, represent a periodic function with period T=1.5 hours. The function corresponding to Figure 1 in fact is given by

$$f(t) = 2\cos\left(\frac{4\pi}{3}t\right) - 2 \quad (1)$$

3. Understanding of Sleep Cycle through Periodic Function

We notice that the values that f (t) takes lies in the interval [–4,0]. That is the range of the sleep stage function S = f (t) described through equation (1) and represented by Figure 1, is any real number from – 4 to 0 both –4 and 0 inclusive.

Now we assign sleep stage S = 0 (Awaken state) and assign each subsequent stage to the successive negative whole numbers. For example, sleep stage 1 will be assigned to S = –1. Now assume t = 0 is when we fell asleep.

The periodic function representing sleep cycle f(t) indicate that we are awaken for every 1.5 hours (sleep stage 0). That is f (1.5) = 0. Since there is one stage for REM category and four stages for Non – REM category our optimal hours of sleep is exactly 1.5 × 5 = 7.5 hours, assuming if you are not waking up in any stage of Non – REM category. But this is impossible. So what would happen if we sleep either less or more than this? Stage 1 sleep is still relatively light sleeping. We now try to determine all values of t for which

$$f(t) \geq -1 \quad (2)$$

To find intervals satisfying (2), it is enough to draw a horizontal line at sleep stage –1. Then all the values of t in our graph for which the graph is above the horizontal line drawn will satisfy (2). We now find the intervals of values of t such that f (t) = –1.

$$2\cos\left(\frac{4\pi}{3}t\right)-2=-1 \Rightarrow \cos\left(\frac{4\pi}{3}t\right)=\frac{1}{2}$$

From this, the possible non-negative values of t are given by

$$\frac{4\pi}{3}t=\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\frac{11\pi}{3},\frac{13\pi}{3},\frac{17\pi}{3},\frac{19\pi}{3},\frac{23\pi}{3},\frac{25\pi}{3},\frac{29\pi}{3},\frac{31\pi}{3},\cdots$$

That is, $t=0.25,1.25,1.75,2.75,3.25,4.25,4.75,5.75,6.25,7.25,7.75,\cdots$ (3)

Conclusion

Thus from (3) the intervals which satisfy (2) are [0,0.25], [1.25,1.75], [2.75,3.25], [4.25,4.75], [5.75,6.25], [7.25,7.75] and so on. From these intervals we observe that except for the first interval if we add 0.25 then the left end-points of the other intervals will be 1.5, 3, 4.5, 6, 7.5 all of which are integral multiples of 1.5. Thus except the first interval, all other intervals are 0.25 hours or 15 minutes away from multiple of 1.5. Thus for every cycle of 1.5 hours in our periodic function described by Figure 1, we miss 15 minutes on endpoints of each interval described above. This shows that we are missing 1.5 hour margin which is the periodicity of the sleep cycle function in Figure 1, by 15 × 2 = 30 minutes for each of the intervals which determine deep sleep with respect to Non – REM category. Hence the average sleep cycle length per period = 1.5 hours = 90 minutes which represent the period of the function and average deep sleep cycle length per period = length of the period – length of missing time for each interval = 1.5 hours – (0.25 + 0.25) hours = 60 minutes. This will make us to understand about our sleep cycle which is an essential aspect for any human to lead healthy and long life.

Dr. P. N. Vijayakumar has more than ten years of teaching experience at Gopalapuram Boys Higher Secondary School, Chennai. He teach Higher Secondary mathematics course. His passion towards making mathematical models and computer designing is laudable. He received Honorary Doctorate from Ballsbridge University in April 2019. He was one of the Trustees in Pie Mathematics Association. He is a regular contributor of mathematics articles and involve very actively in almost all the programmes of Pie Mathematics Association. He offer help in all possible ways for the development of mathematics.

Basel Problem

V.R. Kalyankumar

Introduction

A classical problem in mathematical analysis known as “The Basel Problem” was first posed by Pietro Mengoli in 1650. It was solved by Leonhard Euler in 1734. Basel problem named after the hometown of Euler, is about finding precise value when reciprocals of the squares of the natural numbers are added up continuously. This problem and the subsequent solution by Euler led to great developments in Number Theory in later years. In this article, we will discuss the problem and derive its solution.

Keywords: Infinite Series, Maclaurin Series Expansion, Infinite Product, Sum of Reciprocals of Squares.

1. Description of the Problem

The Basel Problem is concerned with finding the value of the following series:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \quad (1)$$

2. Solving the Problem

From the Maclaurin series expansion for a function which is continuously differentiable at $x = 0$, we have

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots \quad (2)$$

Euler considered f(x) = sin x. Using (2), we get

$$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \quad (3)$$

Dividing the whole equation by x we get

$$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots \quad (4)$$

We are now interested to determine values of x such that $\sin x = 0$

This will happen if $x = n\pi$, where n is an integer.

Hence we can express $\sin x$ as an infinite product given by

$$\sin x = x(x-\pi)(x+\pi)(x-2\pi)(x+3\pi)(x-3\pi)\cdots(x+n\pi)(x-n\pi)\;\cdots$$

Hence if $\frac{\sin x}{x}=0$ **where** $x \neq 0$ **then**

$$\left(1-\frac{x}{\pi}\right)\left(1+\frac{x}{\pi}\right)\left(1-\frac{x}{2\pi}\right)\left(1+\frac{x}{2\pi}\right)\cdots\left(1-\frac{x}{n\pi}\right)\left(1+\frac{x}{n\pi}\right)\cdots=0$$

$$\left(1-\frac{x^2}{\pi^2}\right)\left(1-\frac{x^2}{2^2\pi^2}\right)\left(1-\frac{x^2}{3^2\pi^2}\right)\cdots\left(1-\frac{x^2}{n^2\pi^2}\right)\cdots=0 \quad (5)$$

Now using the condition that $\frac{\sin x}{x}=0$ **where** $x \neq 0$ **from equation (4) we get**

$$1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\cdots=0 \quad (6)$$

Conclusion

From equations (5) and (6), we get

$$1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\cdots=\left(1-\frac{x^2}{\pi^2}\right)\left(1-\frac{x^2}{2^2\pi^2}\right)\left(1-\frac{x^2}{3^2\pi^2}\right)\cdots\left(1-\frac{x^2}{n^2\pi^2}\right)\cdots$$

Now equating coefficients of x^2 **on either sides, we get** $-\frac{1}{3!}=-\frac{1}{\pi^2}-\frac{1}{2^2\pi^2}-\frac{1}{3^2\pi^2}-\cdots$

Hence, we get $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6} \quad (7)$

Equation (7) provides us with the required result namely that the sum of reciprocals of squares of natural numbers is $\frac{\pi^2}{6}$

Editorial Note:

The solution to the Basel Problem by Euler was considered to be one of classic result in mathematical analysis. Though the way Euler arrived at the solution of Basel's problem is not very rigorous compared to modern standards, nevertheless the ingenuity in deriving this solution remains one of the best methods in the history of mathematics. In fact, this solution led Euler to investigate to extend further and introduce what is called "Euler Zeta Function", through which he found a remarkable way of determining sum of reciprocals of even powers of natural numbers and provided a compact formula for the same. This idea was later generalized by German mathematician Bernhard Riemann to introduce "Riemann Zeta Function" which deals with complex numbers rather than real numbers. These works by Euler and Riemann was the beginning of modern Analytic Number Theory.

V.R. Kalyankumar is an active mathematics enthusiast having 13 years of experience in IT industry. Though software engineer by profession, because of the passion towards knowing mathematics, he completed his post graduation in mathematics. His hobbies are reading books on mathematical history and watching documentaries on world history. He got motivated by Pie Mathematics Association to contribute more towards the popularizataion of mathematics.

Triskaidekaphobia

V. Balaji

Introduction

The year 2020 marks the 13th year completion of Pie Mathematics Association which is an NGO in the form of charitable trust functioning exclusively for the propagation of mathematics. Commemorating this event, I wish to present an article about the number 13 explaining its significance and importance. 13 is the number most feared by many throughout the world. 13 is also regarded as the unlucky number by many. In fact, there is a special term attached to the fear of number 13 in the name "Triskaidekaphobia", the title of this article.

Keywords: Triskaidekaphobia, Paraskavidekatriaphobia, Reversal, Prime Numbers.

1. Etymology

I begin by explaining the meaning of the term "Triskaidekaphobia" to have better understanding. Let us split the whole word "Triskaidekaphobia" as parts like the following

Tris	– 3
Kai	– plus (addition)
Deka	– 10
Phobia	– fear

Thus "Triskaidekaphobia" literally means 'The fear of number 13". In fact, this fear is seen to be more if 13^{th} occurs on Friday of some month in a particular year.

The fear of "Friday occurring on 13 th" is called "Paraskavidekatriaphobia". There is a successful Hollywood thriller film based on this fear known as \`Friday The $13^{th}$ \`.

2. Mathematical Properties of the number 13

The number 13 is widely considered as the most unlucky and scariest number in our society. But when we analyze the number mathematically, it turns out to be one of the most interesting and amazing number.

In Mathematics, prime numbers always occupies a special place in the eyes of mathematicians. The number 13 happens to be the 6^{th} prime number in the set of all prime numbers. I shall try to explain some unique and interesting mathematical properties of the number 13 in the following section.

3. Mathematical Properties of 13

3.1 When we square the number 13, we get $13^2 = 169$. If we now reverse the number 13, we get 31, squaring this we get $31^2 = 961$, which is the reversal of square of 13.

3.2 Squaring 13, we get $13^2 = 169$.
If we now insert the + sign between the digits on both sides of this equation, surprisingly it becomes equal on both sides.
For example, $(1+3)^2 = 1+6+9 = 16$.
The same property applies for reversal of 13 namely 31 too.

3.3 Squaring 13, we get $13^2 = 169$.
We split the digits of 169 in two blocks as 16 and 9. We notice that both 16 and 9 are square numbers.
If we add 16 and 9, we get $16+9 = 25$ which is 5^2
If we multiply 16 and 9 we get $16 \times 9 = 144$ which is 12^2
$169 = 25 + 144$
$13^2 = 5^2 + 12^2$.
Notice that the numbers 5, 12, 13 form an Pythagorean triple since they satisfy Pythagoras Theorem.

3.4 We can express the square of 13 namely 169 using the blocks of digits 16 and 9 in a interesting way as $169 = (16 + 9) + (16 \times 9)$.

3.5 When we add all prime numbers up to 13 we get 13^{th} prime number in the sequence.
$2+3+5+7+11+13= 41$, is the 13^{th} prime number in the list of prime numbers.

3.6 When we form a number by beginning with 13 and descending by one continuously until we reach 1 and then continue increasing by one until 13, we get a prime number.
For example, 1312111098765432123456789101112l3 is a prime number.

3.7 When we form a number by concatenating the digits of cubes of natural numbers starting from 13 up to 1 in descending order we get a prime number.
For example, 2197172813311000729512343216125642781 is a prime number.

3.8 The reciprocal of 13 namely 1/13 possess a very interesting property.

$$\frac{1}{13} = 0.076923076923076923 \ldots$$

The blocks of digits 076923 repeat infinitely often. Hence 1/13 is a rational number which is non-terminating and recurring decimal number.

When we multiply 76923 by successive multiples of 13 we obtain following set of interesting equations

76923×13=0999999
76923×26=1999998
76923×39=2999997
76923×52=3999996
76923×65=4999995
76923×78=5999994
76923×91=6999993
76923×104=7999992
76923×117=8999991
76923×130=9999990

Notice that all the answers contain block of five 9's in the middle and the first and last digits form the sequence of ascending and descending order of the first 10 whole numbers. So the sum of first and last digit in each case is 9.

3.9 The first official book in mathematics is called "The Elements". This book was compiled by the Great Greek Geometer Euclid and it consists of 13 volumes.

3.10 The greatest scientist of Antiquity Archimedes proved that there are only 13 possible semi-regular solids that exist. These solids were now called "Archimedean Solids" in the honour of its discoverer.

3.11 13 is the first prime number to obey the following property

$$\frac{2^6 + 1^6}{2^2 + 1^2} = \frac{64 + 1}{4 + 1} = \frac{65}{5} = 13$$

3.12 13 is the 5th lucky number, 6th prime number and 7th Fibonacci number.

3.13 Numbers of the form $n^2 + (n-1)^2$ are called as "Centered Square Numbers". The first six centred square numbers are 1,5,13.25,41,61. We notice that 13 is the third "Centered Square Number". Centered square number is the sum of two consecutive square numbers. Hence 13 can be expressed as sum of two squares.

3.14 $13^2 = 169$ can be expressed as the sum of 1, 2, 3, 4, 5, 6 distinct squares.

3.15 Finally, when we write the word pi, the letter p occurs as 16th English alphabet and letter i occur as 9th alphabet. Joining these two numbers together we get $169 = 13^2$. Thus the word 'pi' has secret connection with the number 13.

Conclusion

As noticed in previous sections, the number 13 has many unique and wonderful mathematical properties.

Acknowledgement

The author acknowledges the help of the books and articles published by Pie Mathematics Association for writing this article.

Editorial Note

Though there is a global belief even today about the fear of number 13 as discussed briefly in this article, in true sense, there is absolutely no need to fear about number 13 at all. 13 is simply a number just like all other numbers having its own characteristics and properties. In fact, no number is harmful or capable of creating ill effects in our lives. All numbers are unique and wonderful in their own way. In this sense, this article provides the chance to dispel the phobia created all along about the number 13 for so many years.

V. Balaji is a mathematics teacher having eight years of experience. He has been a member of Pie Mathematics Association for nearly a decade. He always tries to inculcate interest and likeness towards learning mathematics among students by using day-to-day life examples, activity based teaching and demonstrating concepts using working models. He was trained by Pie Mathematics Association and had provided many lectures and demonstrations at various places. His favourite topic in mathematics is number theory.

Tools of Construction

S. Manooj Kumar

Introduction

The concept of sets has been very fundamental in building many mathematical structures. Sets are as tricky as numbers themselves. In this article, I shall define a set and provide nice real life examples to understand what really a set is. The study of set theory has been a great discussion in early part of twentieth century when several mathematicians proposed curious paradoxes questioning the fundamental assumptions of making sets. There were also many interesting but not so well known conjectures about sets that exist in mathematics. I have presented one interesting conjecture and leave readers to think about it.

Keywords: Well defined, Finite set, Infinite set, Union-Closed.

1. Definition

A set is defined as a collection of well defined objects.

The term "well defined" plays an important role in the definition. For example, the collection of all good people in a certain town does not constitute a set because the term "good people" is not well defined.

2. Some Examples

We also notice that set constitute of objects with some common property between them. For example look the image. It consists of objects which we wear: shoes, socks, pant and so on. I'm sure you could come up with at least a dozen such objects. This should be common to all human beings, because they wear these things. The items we wear in this case are called objects. Hence collection of all such objects forms what is called a 'set'.

Another example for a set is types of fingers that normal humans possess. This set includes five objects namely Thumb, Index, Middle, Ring and Pinky. So a set is a collection of objects grouped together with a certain property in common. Since the fingers in hand are common to all people, type of fingers form objects of a set.

3. Category of Sets

The number of elements present in a set is called its "Cardinality". If the cardinality is finite, then the set is called a finite set otherwise it is a infinite set.

For example, the set of all first fifty square numbers constitute a finite set, where as the set of all prime numbers is an infinite set. In day to day life situations we often consider finite sets. Infinite sets usually occur in nature and they are very important for building mathematical structures like Metric Spaces, Vector Spaces and Topological Spaces.

It will be an interesting exercise to ask students to provide practical examples of finite and infinite sets. Usually they come up with surprising answers. Just like numbers, it is also quite interesting to note that there is no larger or smaller set that can possibly be constructed.

4. The Union Closed Sets Conjecture

A family of sets is said to be union-closed if, given any two sets in the family, their union is as well in the family. Here's an example: {{ }, {1,3}, {2}, {1,2,3}, {1,2,3,4}}.

Take union of any two of those sets and you'll get a member of the same family. Now, provided our family is finite and consists of more than just the empty set, is there always an element that's present in at least half of the sets? In the example above, the answer is yes: The element 2 appears in three of the five sets.

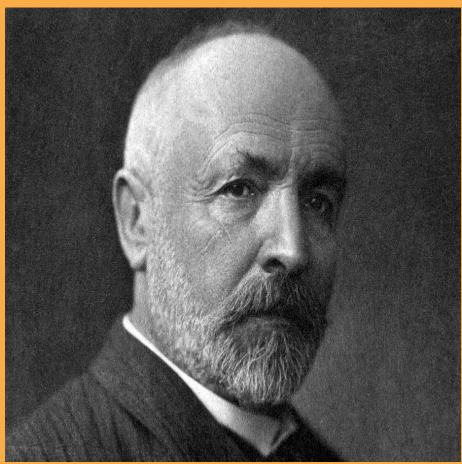
Will this always be the case? Strange to say, no one knows. Though the problem is almost absurdly simple to state, it has remained unsolved since Péter Frankl first posed it in 1979. Henning Bruhn and Oliver Schaudt survey the state of the inquiry here. "The union-closed sets conjecture still has a bit of a journey ahead of it," they conclude. "We hope it will be an exciting trip." Readers may try to think about this and see if you can make any progress in this conjecture.

CONCLUSION

Sets are the fundamental tools in mathematics used for developing higher mathematical concepts like Graph Theory, Abstract Algebra, Real Analysis, Complex Analysis, Linear Algebra, Number Theory and many more. In this sense, sets are considered as building blocks of mathematical structures resembling atoms which make up a whole molecule. Now as a word of caution, sets, by themselves, seem pretty pointless. But it's only when we apply sets in different situations do they become the powerful building block of mathematics that they are. Sets are very widely used both in practical life situations and scientific investigations. In this sense, sets become indispensable tool for us to explore higher mathematical concepts.

Editorial Note

German Mathematician Georg Cantor is considered as "Father of Set Theory" for discovering deep ideas regarding sets



Georg Cantor

S. Manooj Kumar a member of Pie mathematics Association was working as a Post Graduate mathematics teacher in SRM Nightingale Matriculation Higher Secondary School. He is a flexible and organized teacher having seven years of teaching experience, with an ability to explain complicated mathematical concepts in an easily understandable manner. His area of interest is applying mathematics in day to day life situations.

Words and Graphs

Dr. D. Yokesh

Introduction

The concept of drawing mathematical graphs forms an important branch of mathematics known as Graph Theory. The great Swiss mathematician Leonhard Euler laid the foundation of this interesting topic by providing solution to the famous Konisberg Bridge Problem. Today, Graph Theory is considered to be one of the most active research topics and its ideas are getting applied very widely in Science and Technology right from addressing basic issues to most complex problems. In this article, I will try to introduce a small concept from Graph Theory which is a kind of play-way method for students. Read and get yourself involved.

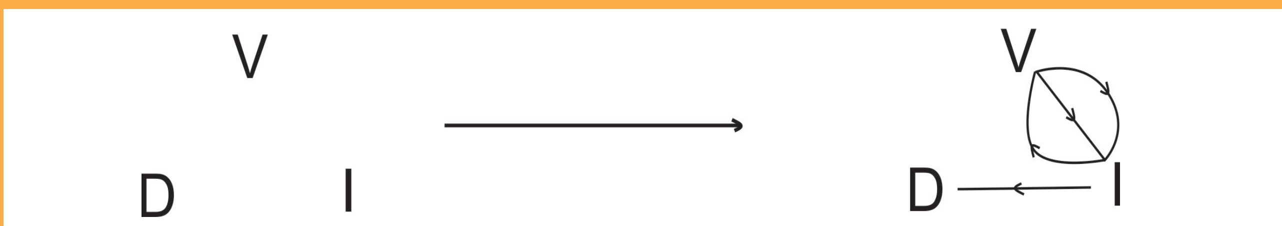
Keywords: Points and Lines, Spelling Net, Non – Planar Graph, Eodermdrome, Complete Graph.

1. Description of the Process

First, we will formally define "Spelling Net".

A Spelling Net is the structure called Graph, constructed when one writes down one instance of each unique letter that appears in a word and then connects these letters with lines, spelling out the word. Each letter of the word is considered as a point or vertex and each line joining those letters is considered as line or edge of the graph.

For example, the spelling net for VIVID is made by writing down the letters V, I, and D and drawing a line from V to I, I to V, V to I, and I to D as shown below.



Different words produce different spelling nets, of course, but every spelling net is an example of a graph, a collection of points connected by lines.

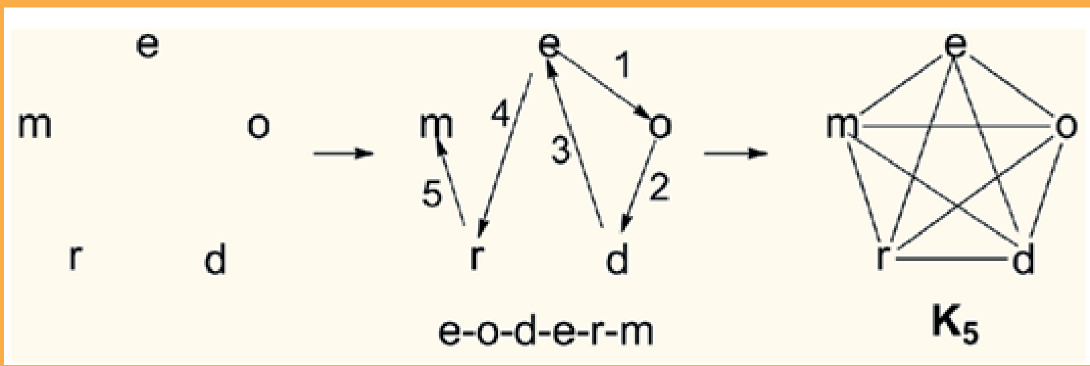
2. Meeting at Non-Vertex

A graph is said to be non-planar if some of the lines cross each other at a non-vertex. In the case of the spelling net, this means that no matter how we arrange the letters, when we connect them in order as describe above, we find that at least two of the lines that cross each other at a point which is not any of the spelling word that we have considered.

If all the lines meet only at the vertex points (that is, at the spelling words) then such a structure is called Planar Graph.

3. Definition

A word with a non-planar spelling net is called an eodermdrome, an ungainly name that itself illustrates the idea. The unique letters in eodermdrome are e, o, d, r, and m. Write these down and run a pen among them, spelling out the word. You'll find that no matter how the letters are arranged, it's never possible to complete the task without at least two of the lines crossing at the non-spelling word namely e, o, d, r, m as shown below:



If you complete drawing all the lines by joining the adjacent letters in our word, you get a graph denoted by K_5 called complete graph on five vertices.

Conclusion

It will be an interesting past time to construct these kinds of graphs for given words. As an exercise, can you find the graphs for the words: (a) INDIA (b) MALAYALAM (c) SUPERSATURATES

Editorial Note:

A great deal of work about EODERMDROME is done by computer scientists Gary S. Bloom, John W. Kennedy, and Peter J. Wexler.

Dr. D. Yokesh has done his research in Graph Theory one of the active research areas of modern mathematics. He possess 15 years of teaching experience at college level and he is currently working as Associate Professor at SMK FOMRA Institute of Technology. He has delivered several lectures and chaired many sessions in mathematics conferences. He has seven research publications to his credit. He is one of the trustees of Pie Mathematics Association.