Mathematical Pixels

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$e^{i\pi}+1=0$

We Serve
Who Deserve

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Dr. R. Sivaraman
Founder, Pie Mathematics Association

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Editor's Foreword

In view of propagating and popularizing mathematics, Pie Mathematics Association (PMA) has been involved in various activities and outreach programmes. The association has been publishing one book per year since its inception in view of reaching several mathematics students, enthusiasts and professionals. In the years 2020 and 2021, when the whole world have to rely on online stream due to pandemic, PMA decided to release e-magazine for meeting its objective of spreading mathematics among everyone. The first issue of Mathematical Pixels e – magazine has been published during Ramanujan's 133rd Birthday Celebrations Anniversary in December 2020. The first issue was released online by most distinguished professor M. Ram Murty, FRS, Queen's University, Canada. Since we received great response towards the first issue, we have planned to bring out two issues per year. Each volume would comprise of two issues and this is second issue of first volume.
I wholeheartedly thank all the contributors which enabled us to bring this magazine.

The purpose of publishing this e–magazine is to encourage and motivate young students and interested teachers who need to know mathematics outside their usual curriculum. I hope that this magazine will satisfy this need. We also encourage everyone interested in mathematics to send their articles for future publication in the mail id piemathematicians@yahoo.com. Depending on the quality and innovativeness, the editorial team may decide to publish. In this second issue, we have published six expository articles by six authors. On behalf of PMA, I thank everyone of them. PMA welcome constructive suggestions for improvement of future issues of the magazine.

Editor
Mathematical Pixels e – Magazine
Pie Mathematics Association

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Associate Professor, Department of Mathematics
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Mobile No: 8608689854

Knowing Natural Numbers

N. Bhagyalakshmi
Mathematics Teacher, Chennai

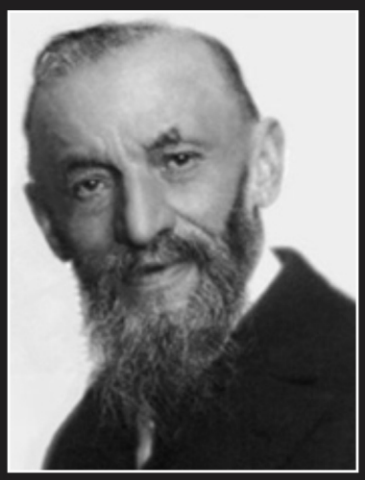
INTRODUCTION

Most of us are familiar with natural numbers also called as counting numbers, as humans of all ages knew how to count like 1, 2, 3, 4, 5, . . .

The very idea of counting itself is very abstract. But do you know how natural numbers are distinct and useful compared to other set of numbers? In this little article you will see the way of constructing natural numbers mathematically.

Keywords: Natural Numbers, Axioms, Successor, Principle of Induction

Idea of Constructing Natural Numbers



Giuseppe Peano

Natural numbers begin with 1 and every natural number after 1, has a predecessor and successor. These ideas seem to be very simple and obvious but honestly it is not so.

The idea of constructing natural numbers itself bears some axioms (axioms are self – evident statements which do not require a proof). These axioms were formulated by Italian mathematician Giuseppe Peano who has authored more than 200 books mainly related to set theory and mathematical logic. He also introduced many notations that were used in current times.

The axioms that Peano presented were called Peano's Axioms for natural numbers. We list those axioms below:

1. 1 is a natural number.
2. 1 is not the successor of any other natural number.
3. Each natural number *n* has a successor.
4. If the successor of *n* equals the successor of *m*, then *n* equals *m*.
5. [The key principle of induction] If a set S of natural numbers contains 1, and if *n* belonging to S implies that the successor of *n* belongs to S, then S contains all of the natural numbers.

Understanding the Axioms

Axiom 2 states that 1 is the first natural number and third states that after 1, every natural number *n* has a successor (it is *n* + 1). The fourth axiom provides the uniqueness of natural numbers. Try to find the reason why axiom 4 is true. The fifth axiom of course, provides conditions for including all natural numbers that we know today.

The five axioms presented above will generate all natural numbers for our mathematical explorations.

Editorial Note:

Can you find successor for rational number say $\frac{1}{2}$?

Can you make axioms similar to the ones presented in this article for real numbers?

N.Bhagyalakshmi
Mathematics Teacher

Ms. N. Bhagyalakshmi, currently working as mathematics teacher at Maharishi School of Excellence Senior Secondary School, Chennai has three years of teaching experience at school level. She has completed her post graduation, Master of Philosophy degrees and had completed B.Ed. course with specialization in mathematics. She is very keen and interested to learn and write about mathematics and she has been involved with activities of Pie Mathematics Association for the past five years.

π

World Pi Day

Since the value of π begins with 3.14... considering the first two decimal digits 14 and integer part 3 of π ,beginning from the year 1988, 14th March of every year is celebrated as World Pi Day. Incidentally, this day also happen to be birthday of one of the greatest physicist of 20th Century, Albert Einstein. On World Pi Day, several events about π were conducted globally.

Viewing Heart Mathematically

R. Sengothai
Mathematics Educator and Writer, Chennai

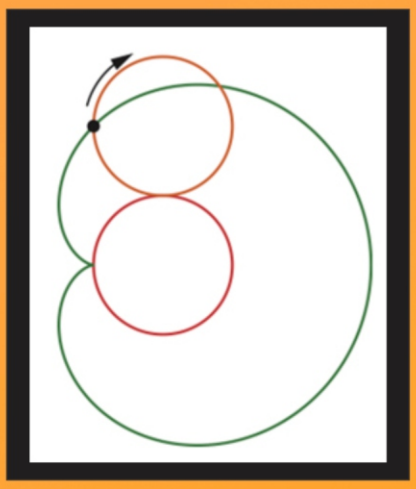
INTRODUCTION

In our day to day life, we often recognize a character of a noble person by remarking "Man of Good Heart". But have you ever thought of the possibility of viewing heart mathematically?

In fact, there is such a possibility, thanks to a curve called "Cardioid". The term "Cardioid" refers to being "Heart Shaped". This is the reason why we call physicians who were specialists in heart surgeries as "Cardiologists". In this article we try to know about this interesting curve.

Keywords: Cardioid, Caustic, Mandelbrot Set, Modular Arithmetic

Creating Cardioid

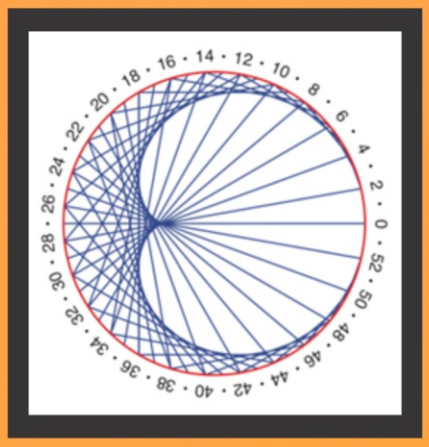


Roll a circle around another circle of the same radius. A marked point on the first circle traces a curve called a Cardioid. (In the figure shown left we have rolled the orange circle around the red circle to draw the green cardioid.) This beautiful heart-shaped curve shows up in some of the most unexpected places. Grab a cup of coffee and we'll show you some.

We do not know for sure, who discovered the cardioid. In 1637 Étienne Pascal—Blaise's father—introduced the relative of the cardioid, the limacon, but not the cardioid itself. Seven decades later, in 1708, Philippe de la Hire computed the length of the cardioid—so perhaps he discovered it. In 1741, Johann Castillon gave the Cardioid its name.

Got your coffee? Turn on the flashlight feature of your phone and shine the light into the cup from the side. The light reflects off the sides of the cup and forms a caustic on the surface of the coffee. This caustic is a Cardioid.

The Mandelbrot set (shown aside) is one of the most beautiful images in all of mathematics. It is the set of complex numbers z, c such that the number $f_c(z)$ does not diverge to infinity under repeated iterations of the function $f_c(z)=z^2+c$. The Mandelbrot set consists of a heart-shaped region with infinitely many circles, spiny antennae, and other heart-shaped regions growing off of it. What shape is that main heart-shaped region? It's a Cardioid.



Cardioids even show up in audio engineering. Sometimes engineers need a uni-directional microphone—one that is very sensitive to sounds directly in front of the microphone and less sensitive to sounds next to or behind it. When they do, they reach for a *cardioid microphone.* The microphone is so-named because the graph of the sensitivity of the microphone in polar coordinates is a Cardioid.

We now generate this fabulous curve through an activity. Begin with a circle (the red circle as shown in the picture attached above). Mark a certain number of evenly spaced points around the circle, N, say, and number them consecutively starting at some point P: 0, 1, 2,..., N-1. Then for each n, draw a line between points n and $2n$ (mod N). In our example, $N = 54$, so we would join points 5 and 10, 19 and 38, and 31 and 8 (since 8 is 62 mod 54). The envelope of these lines is a cardioid. Similarly, there are varieties of other ways that we can consider upon generating cardioid curves.

As an interesting project, readers can take big circle and take for example say $N = 60$ or $N = 80$ points and draw such similar curves which will eventually generate cardioids. Having these many properties, it is no wonder, that cardioid resembles the most vital part of the human body.

R.Sengothai
Mathematics Educator and Writer,Chennai

R. Sengothai, a enthusiastic mathematics teacher and writer is member of Pie Mathematics Association for more than 10 years. Apart from teaching, she writes articles for young students and her works have got published in Tamil Hindu, Dinamalar Pattam newspapers regularly. She has written two books in mathematics the latest being about life and work of women mathematicians released by Pie Mathematics Association during March, 2020. Educating mathematics to young and poor students has been her life ambition. She has written few research papers and had provided some lectures as part of programmes conducted by Pie Mathematics Association.

Pretending the Truth

J. Sangeetha
Mathematics Instructor and Content Developer, Chennai

INTRODUCTION

Teaching mathematics to young children is most challenging as well as provide enormous satisfaction especially, if we make them understand the underlying concepts involved. In my career as a person who strive to remove mathematics phobia among young students, I make sure that they enjoy learning mathematics and consider it a worthwhile experience in life. In this view, I share the following article regarding a simple experiment conducted by psychologist Alan Leslie, reflecting the mindset of young students.

Keywords: Experiment, Empty Cup, Children, Pretence

In a 1988 experiment with 2-year-olds, psychologist Alan Leslie asked each child .to "fill" two toy cups with imaginary "juice" or "tea" from a bottle. Leslie then said, "Watch this!", upended one of the cups, shook it, and replaced it next to the other cup. Then he asked the child to point to the "full cup" and the "empty cup." Though both cups had been empty throughout, all 10 of the 10 students indicated that the "empty" cup was the one that had been inverted.

"This leads to pretending something that is true, namely, that the empty cup is empty," Leslie wrote. "At first glance, this may seem ridiculous. But there is, of course, an important difference between *the empty cup is empty* and *pretending (of) the empty cup 'it is empty.'*" Children distinguish between pretense and reality even when the *content* of those beliefs is the same.This simple experiment help us realize that, far from being unusual and esoteric, cases of 'non-counterfactual pretence', that is, pretending something is true when it is true, are ubiquitous in young children's pretence and indeed has an indispensable role in the child's ability to elaborate pretend scenarios.

Source:

"Pretending and Believing: Issues in the Theory of ToMM," *Cognition on Cognition* [1995], 193-220 by Alan M. Leslie

J.Sangeetha
Mathematics Instructor and
Content Developer

Ms. J. Sangeetha, did her post graduation in mathematics and has been a subject matter expert, instructional designer and trainer involved in guiding and mentoring K-12 content developers and contributing towards mathematics education for nearly 15 years. She has been one of the trustees of Pie Mathematics Association during its inception between the period from 2007 to 2010. She has been involved in conducting various workshops to teachers on best instructional strategies in classroom delivery. She has also been instrumental in designing events that kindles interest in students towards learning mathematics.

Bubble Geometry

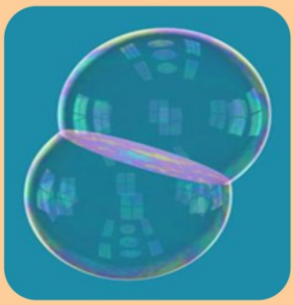
Dr.S. Alamelu, Associate Professor of Mathematics
S.D.N.B Vaishnav College, Chromepet, Chennai

INTRODUCTION

Bubbles are beautiful and fascinating. When they float in air, not only children get excited but most of men and women will also enjoy seeing them. Bubbles have so much of science behind it. It is because of those properties inherited by them, they behave so beautifully. In this short article, you can understand the basic geometry behind bubbles.

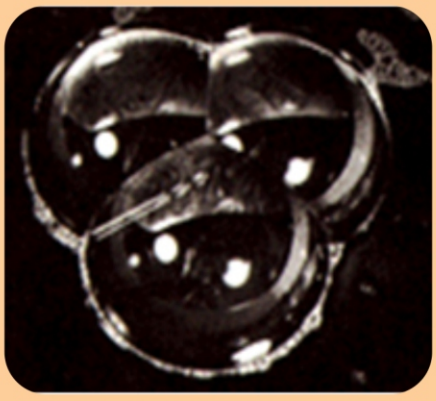
Keywords: Surface Area, Angles, Bubble Walls, Beehives

Intersection of Two Bubbles

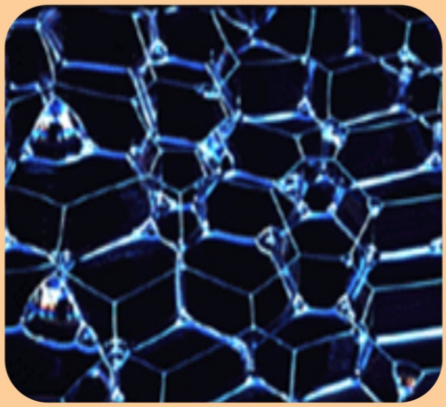


When one bubble meets with another, the resulting union is always one of total sharing and compromise (Human beings could learn a lot from bubbles.) Since bubbles always try to minimize surface area, two bubbles will merge to share a common wall. If the bubbles are the same size as shown above, this wall will be flat. If the bubbles are different sized, the smaller bubble, which always has a higher internal pressure, will bulge into the larger bubble.

Intersection of Three Bubbles



When three bubbles meet, regardless of their relative sizes, the bubbles will meet the common wall at an angle of 120 degrees. This is easy to see in the bubble picture shown above. All three bubbles meet at the center at an angle of 120 degrees. Although the mathematics to prove this are beyond the scope of this article, the 120 degree rule always holds, even with complex bubble collections like a foam.



If you take two sheets of clear glass or plastic separated by about one-half inch, soak them in soapy solution and then blow bubbles between the sheets, you will get many bubble walls. If you look closely, you will notice that all of the vertices where three bubble walls meet (and there are always three) form 120 degree angles. If your bubbles are of uniform size, you will notice that the cells form hexagons and start to look much like the cells of a beehive. Bees, like bubbles, try to be as efficient as possible when making the comb. They want to use the minimum possible amount of wax to get the job done. Hexagonal cells are the tools to do it. But one wonders how Bees know this fact, since they had been constructing honeycombs with hexagonal cells for more nearly millions of years.

Dr.S.Alamelu
Associate Professor Of Mathematics

Dr. S. Alamelu, currently working as Associate Professor in Department of Mathematics at Shrimathi Devkunvar Nanalal Bhatt Vaishnav College for Women (SDNB Vaishnav College), Chromepet, Chennai has more than two decades of teaching experience. Her area of research is Mathematical Modeling and she had been involved in several activities for propagating mathematics in all possible ways. She has published 10 research papers in reputed peer reviewed journals and had presented five papers in various conferences. She has been a regular reviewer of mathematics papers and had been chosen as one of the top reviewer in Publons research database. She has involved in several academic activities related to teaching and evaluation for various courses. She has been a member of Pie Mathematics Association for more than a decade.

Elegant Proof

M. Devika, Associate Professor of Mathematics
Dwaraka Doss Goverdhan Doss Vaishnav College, Chennai

INTRODUCTION

In this little article, I will provide an elegant proof of the fact that *e* is not rational. The number *e* which is referred as base of natural logarithm, is one of the most important real numbers occurring in various possible places quite unexpectedly. For example, it occurs in Probability theory in Poisson Distribution, Normal Distribution, Exponential Distribution to name a few. Similarly in mathematics of compound interest computations, *e* plays a crucial role. Having such an importance, it is worthwhile and exciting to prove interesting fact about *e*.

Keywords: Rational, Irrational, Integer, Proof by Contradiction

Rational Number

Before I formally prove a significant theorem regarding the number *e*, first I will define a rational number formally. A number of the form $\frac{p}{q}$ where *p*, *q* are integers and $q \neq 0$ is defined as a rational number. If the greatest common divisor of *p* and *q* is 1, then we say that the rational number $\frac{p}{q}$ is in its lowest form.

Any real number which is not rational is defined as an irrational number. We are now ready to witness a beautiful result.

Theorem

The number *e* is not rational.

Proof: To prove this, let us use the proof by contradiction method. That is, let us assume that *e* is rational. Then by definition of a rational number, we see that $e = \frac{p}{q}$ where *p*, *q* are integers and $q \neq 0$

In view of exponential series expansion at $x = 1$, we find that

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!} \quad (1)$$

We can now rewrite equation (1) as

$$e = \frac{p}{q} = \sum_{n=0}^{\infty} \frac{1}{n!} = \sum_{n=0}^{q} \frac{1}{n!} + \sum_{n=q+1}^{\infty} \frac{1}{n!} \quad (2)$$

Re-arranging the terms of (2) and multiplying throughout by ***q!*** and simplifying using the definition of factorial function, we get

$$p\times(q-1)!-\sum_{n=0}^{q}\frac{q!}{n!}=\frac{q!}{(q+1)!}+\frac{q!}{(q+2)!}+\frac{q!}{(q+3)!}+\cdots$$

$$=\frac{1}{q+1}+\frac{1}{(q+1)(q+2)}+\frac{1}{(q+1)(q+2)(q+3)}+\cdots$$

$$<\frac{1}{q+1}+\frac{1}{(q+1)^2}+\frac{1}{(q+1)^3}+\cdots=\frac{\frac{1}{q+1}}{1-\left(\frac{1}{q+1}\right)}=\frac{1}{q} \quad (3)$$

Now we observe that the left hand side of (3) is an integer, whereas, the right hand side is not.

In fact, the right hand side quantity $\frac{1}{q}$ is a real number between 0 and 1. This is not possible.

We have thus arrived at a contradiction.

This contradiction proves that *e* is rational is false. Hence, *e* is not rational, which is same as *e* is irrational.

Editorial Note:

The elegant proof presented above appeared in wonderful book written by Hardy and Wright and published in 1960.

M.Devika
Associate Professor of Mathematics

Ms. M. Devika, currently working as Associate Professor of Mathematics in Dwaraka Doss Goverdhan Doss Vaishnav College at Chennai has more than two and half decades of teaching experience at college level. She obtained her M.Sc and M.Phil degrees in Mathematics from University of Madras. 11 students has successfully completed their M.Phil. degree under her supervision. Functional Analysis, Number Theory, Cryptography and Fluid Dynamics were her areas of specializations She has been a Convenor / Organising Secretary for Workshop / Faculty Development Programme / Conference / Webinar conducted by post graduate and research department of mathematics of Dwaraka Doss Goverdhan Doss Vaishnav College, Chennai.